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PART I - EFFECTS OF DIAMETER ON ACTIVE NOISE CONTROL IN RECTANGULAR AND ROUND DUCTS

by

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ABSTRACT

Active noise control (ANC) is particularly useful in hard-walled ducts where plane waves propagate. Higher order mode waves are much more difficult to control. Basic acoustic principles dictate that the cut-on frequency at which higher order modes will first begin to eclipse simple plane waves in a duct will be determined by the cross-sectional diameter of the duct. The lowest frequency for higher order modes will increase as duct diameter decreases. Therefore, the range of frequencies where plane waves dominate will be greater and effective control using ANC better as duct diameter decreases. The result is that somewhat higher frequencies can be controlled with ANC for smaller diameters. Below the first higher order mode cut-on frequency for the largest size studied, there should be little difference in ANC effectiveness between the duct sizes. To test those suppositions, a commercially-available ANC system was used to reduce random noise in rectangular and round ducts having different diameters. Results showed that insertion loss (IL) ranged from 5 to 30 dB in frequencies ranging from 40-1000 Hz, and varied inversely with size as expected. There was no difference in IL below 280 Hz ($p=0.7751$) between the different diameter ducts. There was a significant difference between duct diameters above 280 Hz ($p<0.0001$). The same tests were conducted on a rectangular duct with one cross-sectional dimension fixed and one varied at seven different sizes. Results showed similar IL from 5 to 30 dB that varied inversely with size. There was no difference in IL below 280 Hz ($p=0.3348$) between the different duct dimensions. There was a significant difference between duct dimensions above 280 Hz ($p=0.0220$).

INTRODUCTION

Noise is a ubiquitous occupational and environmental hazard. While personal protective equipment (hearing protection) can control the hazard, some researchers indicate that there are strong workplace cultural factors that preclude their proper use (Royster and Royster, 2000; Patel et al., 2001). It is therefore more effective to devise engineering controls for noise hazards. Engineering controls for noise can be divided into active and passive methods. Passive methods reduce noise by either transmission loss through dense materials or by absorption of reflected waves, or both. Passive controls work better for frequencies at or above 1000 Hz (Driscoll and Royster, 2000).

Active methods use an out-of-phase countering signal to reduce the offending noise. The actual active noise control (ANC) insertion loss (IL) was initially presumed to be through destructive interference of the primary source wave with the secondary out-of-phase control wave. However, it may also be that the primary wave is cancelled by its own reflection from the change in impedance, or resistance to flow, directly in front of the secondary control speaker. The impedance change comes from the speaker itself causing pressure fluctuations as it vibrates. When an acoustic wave encounters any pressure change, such as at the end of a duct or a change in diameter, part of the wave reflects. Additionally, reduction of the primary wave may be a result of the secondary source "unloading" the primary noise source by changing its radiation impedance, consequently reducing the overall sound power output (Snyder and Hansen, 1989; Snyder and Tanaka, 1993).

In any event, ANC works better at frequencies below 500 Hz (Hansen 2001) than for higher frequencies. Compared to passive controls, ANC also has the attractive feature of being less bulky and offering less additional resistance to air flow in duct applications. However, unlike passive controls, for which dimensions are not necessarily important to effectiveness, the success of ANC methods is dependent on the physical dimensions of the space in which noise is to be controlled. ANC is easiest to apply and generally produces the greatest degree of noise reduction in closed channels of limited cross-section (Hansen, 2001). These conditions are present in ducts, so it is not surprising that most examples of successful ANC applications are in noisy ducts (Gordon and Vining, 1992; Pelton et al., 1994; Driscoll and Royster, 2000; Hansen, 2001). A very common field application is reducing noise escaping from exhaust stacks, especially for cases where the main goal is to reduce fan noise contributions to community noise (Gordon and Vining, 1992). It is also used for cases where the goal is to reduce HVAC system noise to increase comfort of building occupants (Pelton et al., 1994). The effectiveness of ANC applications depends heavily on (a) the ability to sample a sound wave at one point and predict what it will be at a later location, and (b) the bandwidth, or how many target frequencies are included in the range. The ability to predict waves depends on the type of wave, while the bandwidth is dependent on the noise source.

Plane waves vs. higher order modes

The reason for the relative effectiveness of ANC methods inside ducts can be better appreciated with an understanding of the different types of sound waves inside a duct. Noise

inside a duct is physically limited in its direction of propagation. Waves can either travel straight down the duct (plane waves) or bounce back and forth between the walls (higher order modes) (Eriksson, 1980; Kino, 1987; Norton and Karczub, 2003). The cross-sectional dimensions of the duct determine where in the frequency spectrum the plane waves will be the dominant contributor to the overall noise, and where the shorter wavelength, higher order modes will become the dominant noise contributor.

Plane waves have a uniform pressure distribution over the cross-section of the duct and propagate down the center of the duct for wavelengths at least twice the cross-sectional diameter of the duct (Bies and Hansen, 2003). Using ANC methods, these plane waves can be analyzed at one location in the duct and countered at some distance away in the same duct with high accuracy (and thus high effectiveness) since their frequency spectra do not change rapidly as they move down the duct.

Shorter wavelengths behave quite differently from wavelengths long enough to be only plane waves. Instead of moving longitudinally in the duct, they bounce back and forth between the duct walls and set up waves corresponding to higher order "modes." A higher order mode is an integer value of reflections representing waves that are repeatedly traveling the same pathway as they reflect between the duct walls.

The boundary or "cut-on" frequency (f_{co}) at which higher order modes begin in round ducts can be estimated using Equation 1 (Bies and Hansen, 2003):

$$N = \left(\frac{f_{co} D}{c}\right)^2 + 1.5\left(\frac{f_{co} D}{c}\right) \qquad \text{1}$$

where

N = integer mode number
f_{co} = cut on frequency (Hz)
D = duct diameter (ft)
c = speed of sound in air (fps)

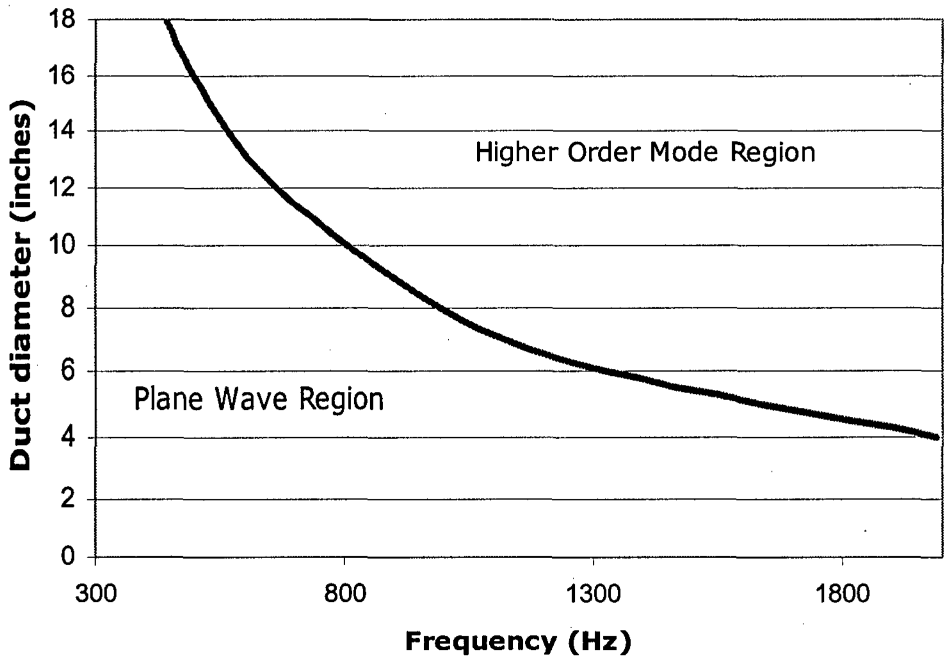

Figure 1. Round duct plane wave vs. higher order mode regions

As can be inferred from Equation 1 and is plotted in Figure 1, the frequency for a given mode number is inversely related to the duct diameter (*D*). This suggests that modes become increasingly important at larger diameters. As one continues up the frequency spectrum for a given diameter, higher order modes will increasingly dominate the noise content. The dominant frequency below which plane waves will still dominate the higher order modes (f_d) can be estimated for round ducts by Equation 2 (Eriksson, 1980; Bies and Hansen, 2003; Norton and Karczub, 2003):

$$f_d = \left(\frac{0.5861}{D}\right)c \qquad (2)$$

where

f_d = dominant frequency (Hz)
D = duct diameter (ft)
c = speed of sound in air (fps)

For rectangular ducts, the cut-on frequency can be estimated from Equation 3 (Norton and Karczub, 2003):

$$f_{co} = \left(\frac{c}{2\pi}\right)\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} \quad \text{..................3}$$

where

f_{co} = cut on frequency (Hz)
c = speed of sound in air (fps)
a = width of duct in x direction
b = width of duct in y direction
m = number of pressure nodal lines in x direction
n = number of pressure nodal lines in y direction

As Equation 3 implies, the plane wave and higher order mode frequency regions depend on both the vertical and horizontal cross-sectional dimensions. If one of the dimensions were held constant, then there would be different cut on frequencies for the first, second, third, etc. modes based on the other cross-sectional dimension. However, the first higher order mode may depend only on the constant dimension if it were larger than the other dimension. Figure 2 displays a plot for a constant vertical dimension of 61 cm and a varied horizontal dimension of less than 61 cm.

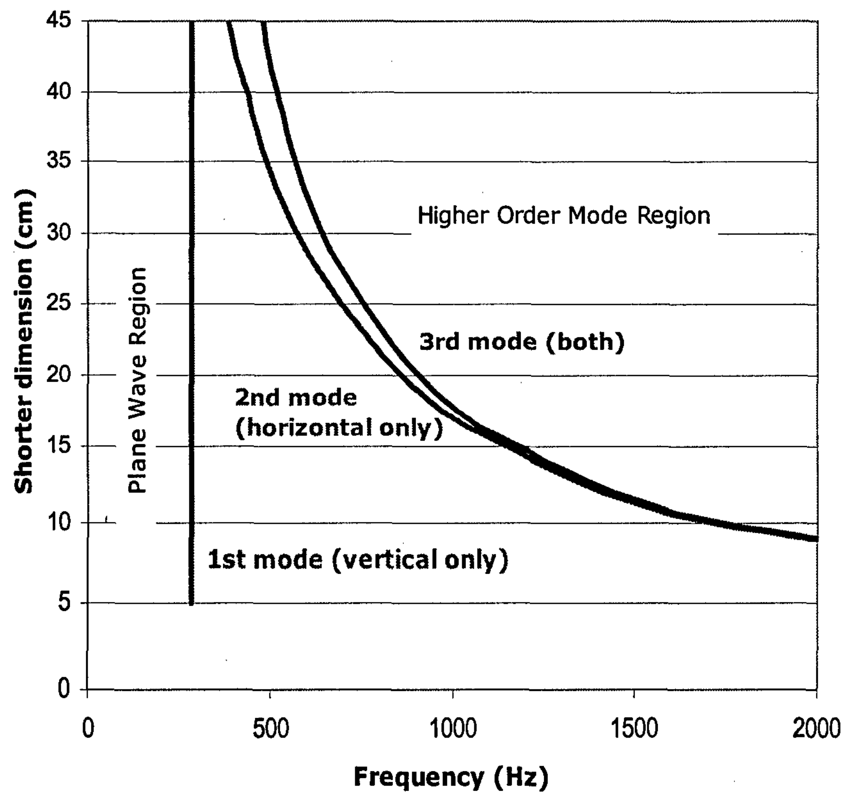

Figure 2. Rectangular duct plane wave vs. higher order mode regions

For a given shorter horizontal dimension, the first lowest frequency mode would be due to the constant larger vertical dimension, then the second mode due to the horizontal only, then the third due to both dimensions. The degree to which the rectangular duct higher order modes contribute to and potentially dominate the noise spectrum is unknown.

Since higher order mode waves bounce back and forth in the duct space as the wave front moves down the duct, the frequency content of the wavefront is difficult to predict at a distant position. Therefore, ANC is much easier to employ against plane waves than higher order modes. Also, since higher order mode cut-on frequencies are inversely related to the cross-sectional dimensions of the duct, smaller ducts have a larger frequency range that is dominated by plane waves and can be more readily controlled by ANC methods. Hence, one could reasonably expect that values of IL achieved at a given frequency at one diameter will fall with larger diameters or rectangular dimensions as planar waves become progressively less important than higher order modes. This is widely assumed to be true by ANC practitioners

(Bies and Hansen, 2003), but the authors were unable to find any published literature relating duct diameters and ANC performance.

Plane waves are simple to control because the movement of the wave down the duct can be easily predicted from the length and the speed of sound. Higher order modes are erratic and therefore difficult to predict and control. As a result many sensors and control speakers are needed to sample and counter higher order modes (Mazanikov et al., 1977; Eriksson et al., 1989; Zander and Hansen, 1992; Pelton et al., 1994). Therefore, if the frequencies involved are largely limited to those producing plane waves, ANC is more likely to produce substantial insertion losses.

Two methods used to reduce duct dimensions without reducing total cross-sectional area are (1) using many smaller ducts for the same volumetric flow as in Figure 3 or (2) using axial vane splitters for cross-sectional partitioning as in Figure 4. It is not always practical to substitute several smaller diameter ducts for a larger one since the pressure due to air flow is proportional to diameter to the 1.2 power (Guffey and Hickey, 1983). (Note that Guffey and Hickey (1983) report that using a single smaller duct with the same flow would change the pressure by the diameter ratio to the 4.5 power.) While splitters have been used to increase surface area of acoustically absorptive material in ducts for some time (Cullum, 1949; Beranek, 1960), there is only one study applying them to active noise control (Eghtesadi et al, 1986). As is discussed next, another primary consideration in applying active noise control is the width of the frequency bands. The amount of insertion loss is limited by the size of the target frequency range. ANC systems can be optimized for only a narrow range of frequencies at a time (Hansen, 2001).

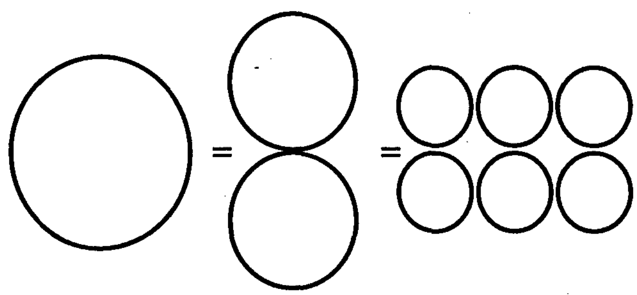

Figure 3. Equivalent volumetric round ducts

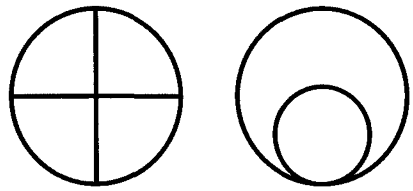

Figure 4. Cross-sectional partioning schemes for round ducts

Broadband random noise vs. tonal noise

Noise problems in the occupational and environmental health (OEH) field are typically broadband (i.e., a broad range of frequencies) and are seldom narrow band ("tonal"). "Pure tones" (single frequency) are rare. Noises in OEH practice also typically are "random" in that amplitudes at each frequency fluctuate separately without following a cyclical or other pattern. Tonal noise tends to vary non-randomly, and pure tones may be quasi-constant. Hence, while the bandwidth of typical noises may be constant, the amplitude of the frequencies inside the band changes constantly. A commonly-encountered source of narrow band noise comes from the repetitive passage of fan blades as a fan wheel rotates. Because of the repetitive, easily predicted nature of pure tones, ANC is extremely effective in reducing them, even the higher order modes. In applying ANC to actual industrial duct noise problems, Bies and Hansen (2003) report that:

> "Typical results achieved are 15-20 dB over two octaves of random noise and 20-30 dB for tonal noise. Typical frequencies which are controlled range from 40 Hz to 400 Hz."

Figure 5 gives a visual representation of potential ANC insertion losses for different bandwidths taken from the words of Bies and Hansen (2003) above. Note that the actual effectiveness at some frequencies is different from that reported by Bies and Hansen (2003). For a given broadband noise, each ANC system uses an algorithm to determine the most important (i.e., highest amplitude) frequencies. The smaller the range of frequencies and the fewer individual frequencies of concern, the more accurately the ANC system can predict and counter those frequencies. Even if insertion loss is desired across a wide range of frequencies,

it is generally most effective to focus on a single two-octave band within the range where ANC has the potential to be effective. If the ANC system can handle several channels independently, the adjacent two-octave bands can be controlled by a separate channel.

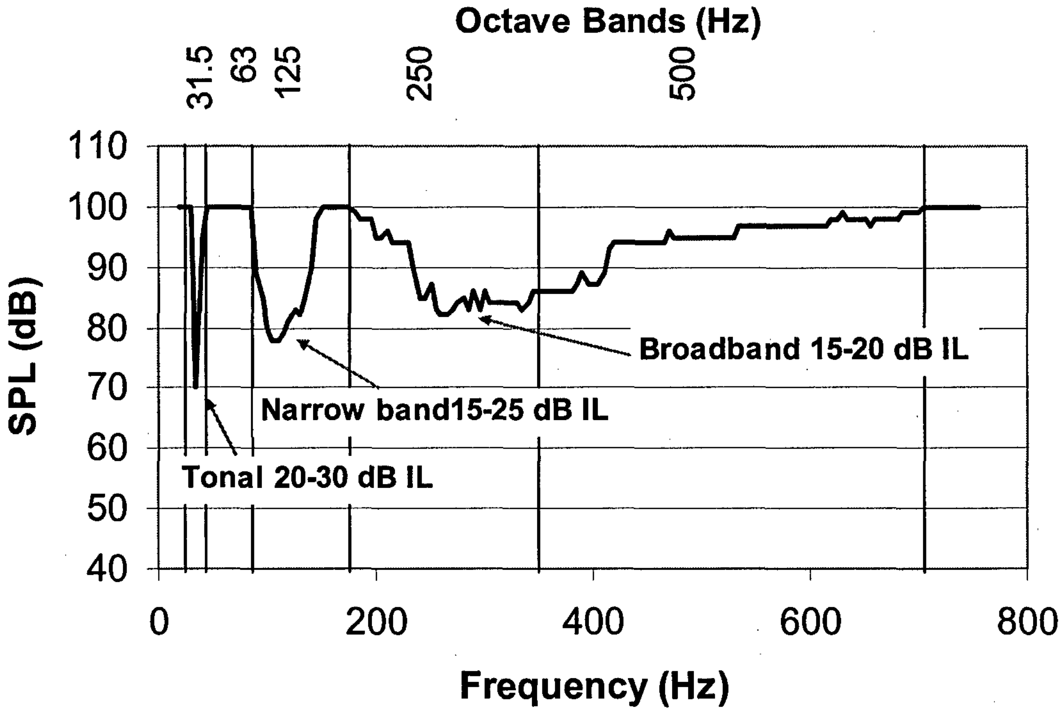

Figure 5. Effects of bandwidth on active noise control insertion loss

Since broadband random noise is much more often encountered than tonal or narrow bands, this study focused on control of broadband random noise only. Also, although duct materials differ in density and transmission loss, durability, vibration transfer, and other important properties, two different duct materials were used for ease of construction. The author could find nothing published to address material specifically. There should be an effect of material due to vibration transfer and transmission loss, but it should be dwarfed by the effect of diameter at frequencies beyond the plane wave dominant frequency (f_d) for that cross-sectional duct dimension. It is not clear whether the results for one material can be applied accurately to other materials.

In spite of the volumes of ANC research, some fundamental questions remain unanswered. While there are equations for estimating the frequency ranges of plane wave and higher order mode dominance in round and rectangular ducts, there have been no empirical studies to confirm the efficacy of the equations. Given that so many researchers state the importance of limiting ANC in ducts to plane wave control, it is rather important to better understand the limits and degree of ANC success with regard to plane waves. Since the duct cross section is the only determining variable besides shape, it would be useful to research the ANC performance on plane waves in round and rectangular ducts of varying cross-sectional dimensions. Information gained would aid in the design of strategies to use cross-sectional partitioning by axial vane splitters in large ducts to extend the frequency range of plane wave dominance and improve ANC of broadband noise in large ducts. After reviewing the literature, no diameter effects studies or cross-sectional partitioning strategy studies were found except Eghtesadi *et al.*'s (1986) study on energy conservation. The data obtained from such experiments would aid in devising simple ANC solutions for large duct broadband noise problems so often encountered in industry.

Problem statement

The research presented here attempted to extend the usefulness of ANC in large ducts by exploring the effects of cross-sectional dimensions on ANC insertion loss in large ducts. The initial information gathered from cross-sectional dimension studies in both rectangular and round ducts was used to help design cross-sectional partitioning experiments that focused only on round ducts that will be published elsewhere.

Common ANC Apparatus

The active noise control test device was common throughout all studies and is discussed in this section. The apparatus difference between the studies was in the different ducts used. The unique apparatuses for each study are presented in the individual sections pertaining to those studies.

The simple active noise control (ANC) system sketched in Figure 6 was used for the experiments described here. The components consisted of a source speaker attached tightly to one end of the various ducts with a directional reference microphone next to the source speaker in the tube. The ¼" array microphone (PCB Piezotronics, Depew, NY) was made

directional by inserting it into a ½" I.D. four ft long X5305 microporous tube (Porex, Atlanta, GA). A directional reference microphone preferentially senses the source speaker wave impinging at the tip and was necessary to prevent feedback from the control speaker noise broadcast at the other end of the duct reaching the reference microphone.

The source speaker signal was a random broadband white noise source driven by a signal generator on an OR-38 (OROS, Falls Church, VA) real-time analyzer (RTA). The reference microphone signal was fed into the EZ-ANC II active noise controller (Causal Systems, Inc., Adelaide, Australia) which used a "filtered-x" control algorithm to determine the signal it generated for the control speaker to counter the noise coming down the duct. Another ¼" array microphone (PCB Piezotronics, Depew, NY) was used as the "error" microphone (see Figure 6) to detect the residual sound after control (i.e., the sound not "cancelled" by the downstream speaker). The active noise controller dynamically adjusted the signal sent to the control speaker to minimize the residual sound. For the experiments, the error microphone signal was split off to the real-time analyzer to provide a result reading (with and without ANC).

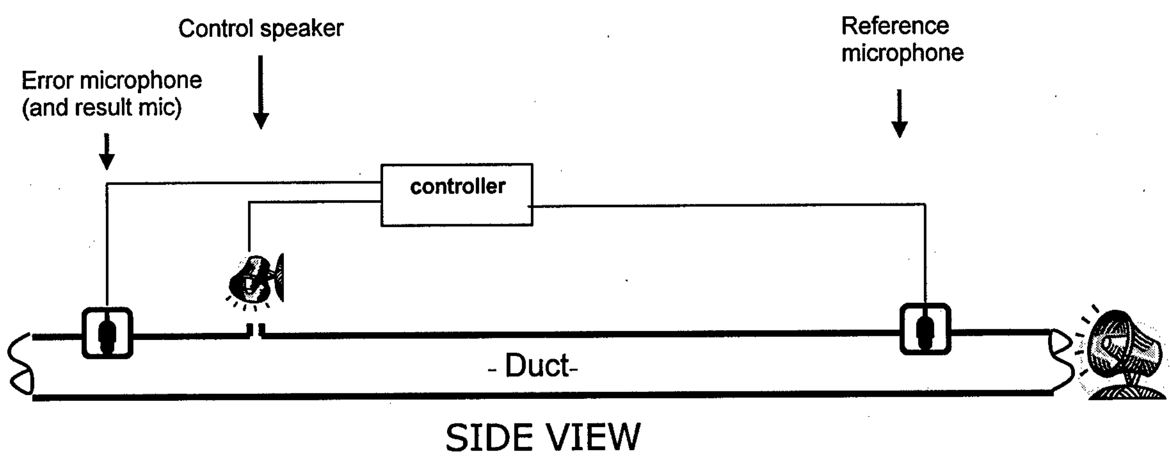

Figure 6. Experimental apparatus

Frequency Focus Bands

As indicated previously, "focus bands" of frequencies aided in comparing the best ANC insertion loss achievable for different frequency regions. Focus bands two octaves wide were developed to use in the experiments (Table 1). The focus band encompassing the 500-

1000 Hz bands was split into single octaves because of the difference in octave width at higher frequencies. Figure 7 provides a visual representation of the width of the frequency focus bands.

Table 1. Range of frequencies used to "focus" ANC controller

Focus Bands	Octave bands included	Frequency range
1	31.5 and 63 Hz	25-90 Hz
2	125 and 250 Hz	90-355 Hz
3	500 and 1k Hz	355-1400 Hz
3a	500 Hz	355-710 Hz
3b	1k Hz	710-1400 Hz

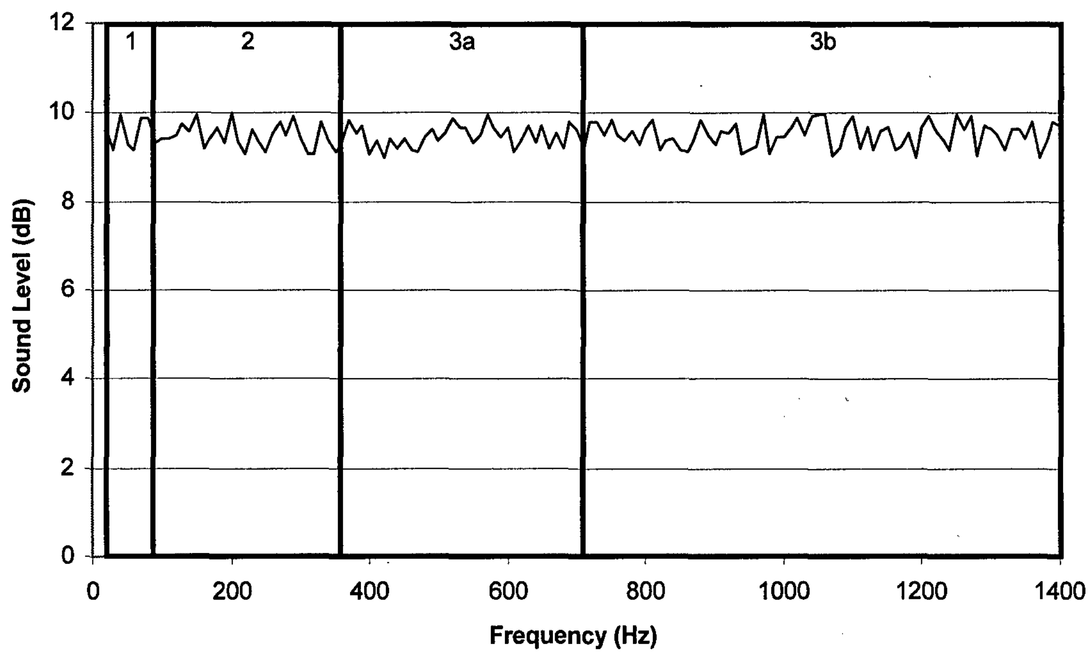

Figure 7. Frequency range of focus bands

A preliminary study was performed using these focus bands to determine the best method of reporting results. Insertion loss estimates from 1/3 octave band readings on various duct sizes were compared between focus bands. For some situations, the IL was larger for a frequency due to a focus band that did not include the frequency due to the inexact nature of sound filters. (e.g., the highest IL for the 500 Hz 1/3 octave band may be when using the 125 and 250 Hz focus bands.)

The insertion loss (ANC off – ANC on) values from all focus bands at each frequency were recorded, and the highest IL estimate taken. This maximum insertion loss (IL_{max}) method can be supported from the idea that the ANC system would be tuned to reduce the noise of highest concern in practice. For those reasons, the IL_{max} method was used as the "best achievable result" when presenting results in this investigation.

A further concern was how to best summarize the results. Since the thrust of all the experiments was to determine the difference in ANC IL above a certain frequency where higher order modes would begin and compare interventions, a common frequency breakpoint of 280 Hz was chosen. The 1/3 octave bands of 20-250 Hz were summed by decibel addition as a low frequency IL_{max} value. The 1/3 octave bands of 315-5,000 Hz were summed as a high frequency IL_{max} value. It should be noted that the high frequency value actually incorporates a range that would be considered "low" or "middle" frequency by practitioners. However, the high frequency value contains the frequency range that is reported as difficult to control by either active or passive controls.

The signal generator for the source speaker used a broadband random white noise focused on the frequency ranges two octave bands wide for the 31.5 and 63 Hz bands, the 125 and 250 Hz bands, and the 500 and 1000 Hz bands. The signal was focused on the single octave bandwidths for the 500 and 1,000 Hz bands, also, since those bands are much wider than the lower frequency bands. The filters on the active noise controller were also set to give priority to the microphone input signals in the set focus bands.

Within each study, the order of trial runs was randomized. The test ducts were dismantled in-between runs, so that even testing the same conditions two times in a row would not be simply "repeated measures." Data was evaluated using JMP Intro software (SAS Institute, Inc., Cary, NC).

STUDY I – DUCT SIZE AND SHAPE

Since the plane wave region can be described in rectangular and round ducts by Equations 1 and 3 as dependent on the cross-sectional dimensions of the duct, a study was designed to ascertain how well these independent variables correlated to the dependent variables of high and low frequency ANC insertion loss. The first study was designed to examine the effect of varying a single dimension of a rectangular duct to change the cut-on

frequency of the first higher order mode. The second study was designed to examine the effect of different round duct diameters used to change the cut-on frequency of the first higher order mode. These effects on the higher order mode cut-on frequency should directly affect the ANC insertion loss that could be achieved.

Variables and Hypotheses

The two dependent variables monitored were the insertion loss at two different frequency ranges. One-third octave band values, with ANC on, were subtracted from the same bands with ANC off to give a measure of insertion loss for each 1/3 octave band. The maximum insertion loss values among the frequency focus bands were recorded for each 1/3 octave band. The insertion loss values were then summed from 20-250 Hz as a low frequency IL_{max} value, and summed from 315-5000 Hz as a high frequency IL_{max} value. These two values, $IL_{max} \leq 250$ Hz and $IL_{max} \geq 315$ Hz, were the dependent variables.

There are numerous independent variables that affect the IL_{max} results for ANC of random noise in ducts. The independent variables of bandwidth, ANC controller operation, and software settings were held constant. Microphone position was held constant during each individual study. The remaining independent variables were: rectangular dimension and round diameter. As these variables could not be tested in the same apparatus, a series of studies was devised to address one independent variable at a time. The studies with variables, null hypotheses, and test type are listed in Table 2.

Table 2. Series of studies to describe independent variables

Study	Variable	H_0	Test
Ia	Rectangular dimension	No effect of horizontal dimension	1-way ANOVA
Ib	Round diameter	No effect of diameter	1-way ANOVA

Test Apparatus

The rectangular duct study was performed with a 24 ft long rectangular duct constructed of 3/8" thick plywood. The duct was constructed of 8 ft lengths connected end

to end and sealed with foam. The cross-sectional height of the duct was fixed at 61 cm, while the width was variable from 7 to 55 cm (Figure 8). One side wall was moveable, with threaded 3/8" steel rods used to fix the side wall at certain widths for the experiments. The reference microphone was suspended by nylon thread in the center of the cross section 1 ft inside the source end of the duct. The error microphone was insulated from vibration inside a foam tube and placed on a wire stand 1 ft inside the far end of the duct, centered in the cross section (see Figure 9).

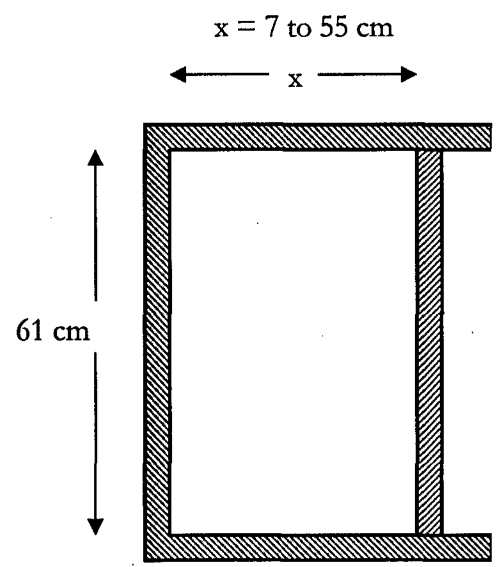

Figure 8. Study Ia Apparatus, Cross-sectional view of rectangular duct apparatus

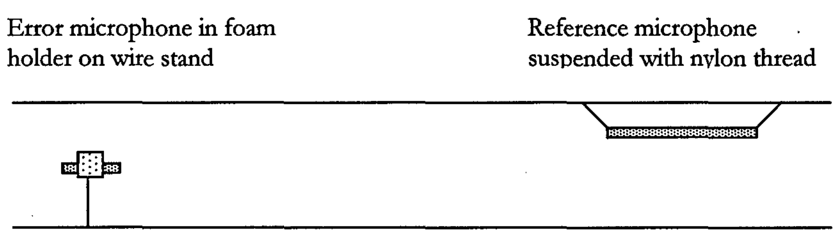

Figure 9. Study Ia Apparatus, Side view of microphone mounting methods

The round duct study tested for the effects of diameter in round ducts using tight-fitting laser-welded center-seam galvanized steel duct (Nordfab, Inc., Thomasville, GA) with diameters 4, 6, 8, 12, and 18 inches. The ducts were selected based on availability and space constraints.

The ducts were all 5 ft long lengths clamped and tightly sealed at the ends to form a 20 ft long straight path. The source speaker was joined to one end of the run of ducts, and the control speaker joined to a 4 inch length of duct connected to the straight duct with a 60-90° junction fitting at a distance of roughly 20 ft from the source speaker at the other end of the run of duct. (See Figure 6) Given the 4 ft porous plastic tube needed to make the reference microphone directional, the actual duct length between the reference microphone and the control speaker was actually 16 ft. Preliminary testing established that 15 feet was sufficient length to allow travel time for the active noise controller processor to determine a counter signal for the noise as the plane wave traveled down the length of the duct.

Rectangular Duct

Rectangular Duct Model and Study Design

Study Ia was designed to test the hypothesis that there was no effect on IL_{max} from changing the cross-sectional width of the rectangular duct while holding the height to 61 cm. Seven cross-sectional widths were selected based on equal 8 cm spacing between the largest and smallest possible widths of the test fixture (7, 15, 23, 31, 39, 47, and 55 cm). A fixed effects model was developed to describe the experiment:

$$IL_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

Where:

IL_{ij} = insertion loss in decibels (dB)
μ = average insertion loss for all treatments
τ_i = effect of the i^{th} cross-sectional width, $i = 1, 2, \ldots, a$
ε_{ij} = random error, $j = 1, 2, \ldots, n$

This model was used to test the following hypothesis on the effect of diameter:

$$H_0: \tau_1 = \tau_2 = \ldots = \tau_a = 0$$

H_1: at least one $\tau_i \neq 0$

Stated non-mathematically, the hypothesis was:

H_0: No effect of cross-sectional width (horizontal dimension)
H_1: At least one cross-sectional width effect different from another

The study design employed the one-way analysis of variance (ANOVA) procedure to analyze the seven levels of the single treatment. Power calculations indicated that, given the average variability in the preliminary tests ($\Sigma \tau_i^2 = 194.9$ and $s = 6.98$ dB), two replicates (n = 2) were required to test the cross-sectional width effect with a = 7 treatment groups with sufficient power ($\beta \leq 0.20$) (Montgomery, 2001). The fourteen runs were randomized to prevent any bias from order or time.

Rectangular Duct Results

The results of the rectangular duct study are shown in Figure 10. The IL_{max} values in decibels for the two dependent variables (low and high frequency) are plotted against the independent variable of cross-sectional width in centimeters. The open circles represent the low frequency data, and the closed triangles represent the high frequency data. Regression lines were added to the figure to help analyze the data.

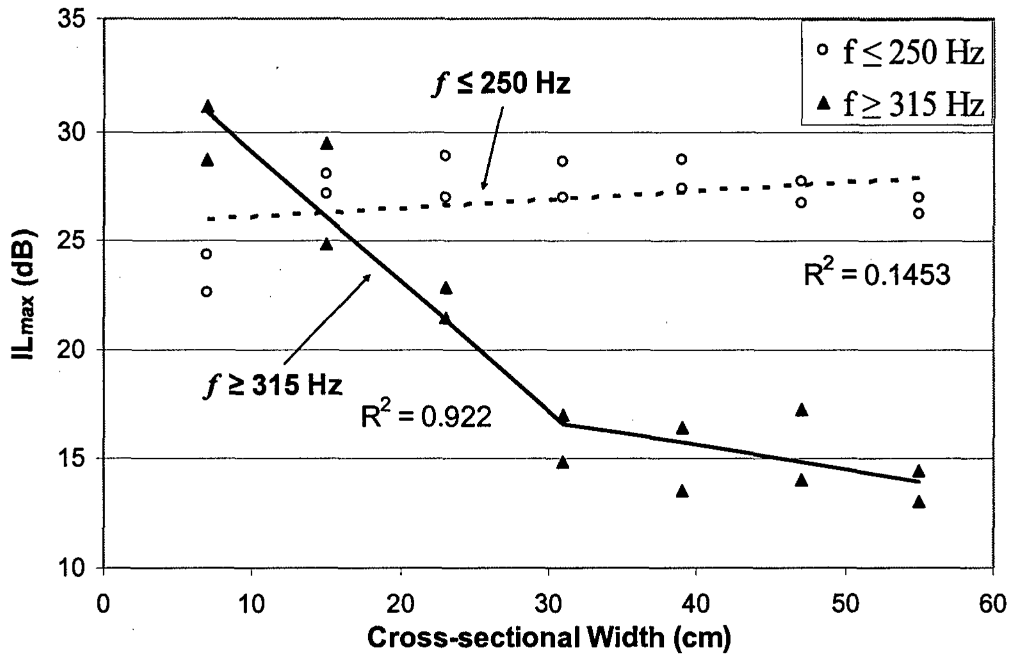

Figure 10. Rectangular Duct Results, IL_{max} by rectangular cross-sectional width

Low Frequency: For the low frequency data, the best fit line does not describe the data well, with an associated R^2 of only 0.1453. However, it appears that the dependent variable does not have a very large effect on the low frequency IL_{max} result. In other words, the IL_{max} is constant at low frequency with regards to cross-sectional width.

The treatment levels appeared to have unequal variances, so O'Brien's test was conducted on the group variances (actually an ANOVA on the variances from the group means) to determine whether they were sufficiently equal (homoscedasticity). The O'Brien test returned a p-value of < 0.0001, so that the treatment variances were considered unequal, which nullifies the usual ANOVA results. Therefore Welch's ANOVA procedure (Welch, 1951; Brown and Forsythe, 1974 and 1974a), which is designed for testing group means when variances are unequal, was conducted on the data. Welch's ANOVA uses the reciprocal of the sample variances of the group means to weight the means for the F-test. Welch's ANOVA returned p=0.3348 for the low frequency data. The null hypothesis that there was no effect from the cross-sectional width on the low frequency data could not be rejected at α=0.05.

High Frequency: The high frequency data were certainly affected by the cross-sectional width. As the width increases, the cut-on frequency of the first higher order mode decreases, so that the overall IL_{max} at the higher frequencies should decrease as well. That is reflected in the data in Figure 12. A linear regression line fit to the high frequency data had an R^2 of 0.8279 and does describe the data relationship somewhat. An exponential line fit to the data only increased the value of R^2 to 0.8401. However, the high frequency data seem to reach a cut-off point with a change of slope between 23 and 31 cm. Therefore, multivariate regression was used to determine a fit for the data in the case of a breakpoint between two slopes. The regression was fit using Equation 4.

$$\hat{Y} = b_0 + b_1 X_1 + b_2 (X_1 - 31\,cm) X_2 \ldots\ldots\ldots\ldots\ldots\ldots 4$$

Where:

$\hat{Y}$ = IL_{max} (dB)
b_0, b_1, and b_2 = Regression coefficients
X_1 = Cross-sectional width of the rectangular duct (cm)
X_2 = 1, when $X_1 > 31$ cm, otherwise $X_2 = 0$

The R^2 value for the multivariate regression line from Equation 4 was 0.922, which described the data well. The high frequency data also failed O'Brien's tests and were considered heteroscedastic ($p < 0.0001$). Welch's ANOVA returned $p=0.0220$ for the high frequency data. The null hypothesis that there was no effect from the cross-sectional width on the low frequency data was rejected for the high frequency data.

Rectangular Duct Discussion

The results of the rectangular duct study support the logic based on the higher order mode cut-on frequency expressed in Equation 3. For low frequencies, there was no effect of changing the cross-sectional width. For high frequencies, there was a significant effect of changing the cross-sectional width, so that IL_{max} decreased with increasing width. There appears to be a limit in the effect somewhere between 23 and 31 cm as the slope of the data changed from negative to flat.

According to Equation 3, the first higher order mode would depend on the larger cross-sectional height, which was fixed at 61 cm. This mode would appear above 282 Hz. Therefore, there should be no difference in IL_{max} below this point regardless of the cross-sectional width, as was seen in the data. However, above 282 Hz, the first mode would begin,

and the second mode, tied to the cross-sectional width, would begin at different points depending on the width (Table 3 and Figure 2).

Table 3. Second higher order mode cut-on frequency by cross-sectional width

Cross-sectional Width (cm)	Second Higher Order Mode Cut-on Frequency (Hz)
7	2460
15	1148
23	749
31	556
39	442
47	366
55	313

Since the cut-on frequency for the second higher order mode decreases with increasing cross-sectional width, the range of frequencies with only plane waves and the first mode also decreases with increasing width. Therefore, the IL_{max} for the high frequency range should be smaller with increasing cross-sectional width, which was seen in the data.

Rectangular Duct Conclusion

For the high frequency IL_{max} data, there was a 6 to 14 dB average increase in insertion loss for successively smaller cross-sectional widths (23, 15, and 7 cm) compared to the 31 cm treatment. There is a significant effect of cross-sectional width on high frequency IL_{max} for rectangular ducts, and the effect follows the logic of the underlying equation.

Round Duct

This set of experiments attempted to describe the effects of round duct diameter on ANC insertion loss. While the only relevant variable according to Equation 1 is the duct diameter, other potential confounders were nevertheless kept constant. For instance, only steel duct of the same type and manufacturer was used in the study. Tight-fitting laser-welded center seam galvanized steel duct (Nordfab, Inc., Thomasville, GA) of diameters 4, 6, 8, 12, and 18 inches was used. Other materials or manufacturing methods were not incorporated to limit confounding.

Round Duct Model and Study Design

Study Ib was designed to test the hypothesis that there was no effect on IL_{max} from changing the diameter of the round duct. Five different diameters were selected based on available sizes (4, 6, 8, 12, and 18 inch). A fixed effects model was developed to describe the experiment:

$$IL_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

Where:

IL_{ij} = insertion loss in decibels (dB)
μ = average insertion loss for all treatments
τ_i = effect of the i^{th} diameter, $i = 1, 2, \ldots, a$
ε_{ij} = random error, $j = 1, 2, \ldots, n$

This model was used to test the following hypotheses on the effect of diameter:

H_0: $\tau_1 = \tau_2 = \ldots = \tau_a = 0$

H_1: at least one $\tau_i \neq 0$

Stated non-mathematically, the hypothesis was:

H_0: No effect of diameter
H_1: At least one diameter different from another

The study design used to analyze the five levels of a single treatment was the one-way analysis of variance (ANOVA) procedure. Power calculations indicated that, given the average variability in the preliminary tests ($\Sigma\tau_i^2$ = 194.9 and s = 2.21 dB), three replicates (n = 3) were required to test the diameter effect with a = 5 treatment groups with sufficient power ($\beta \leq$ 0.20) (Montgomery, 2001). The fifteen runs were randomized to prevent any bias from order or time.

Round Duct Results

The results of the round duct study are shown in Figure 11. The IL_{max} values in decibels for the two dependent variables (low and high frequency) are plotted against the independent variable of duct diameter in inches. The open boxes represent the low frequency

data, and the closed triangles represent the high frequency data. Linear regression lines were added to the figure to help analyze the data.

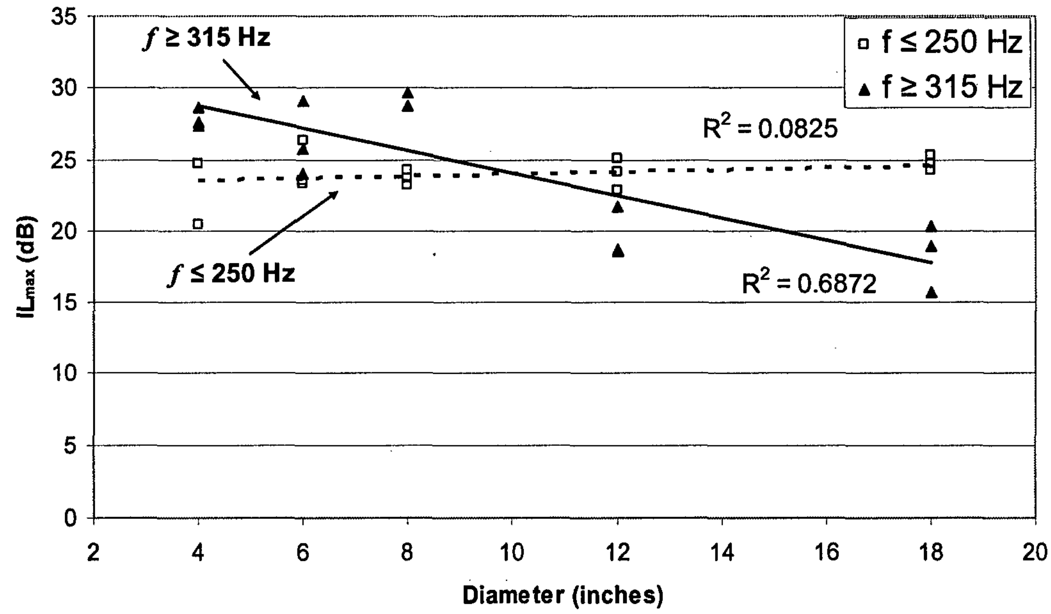

Figure 11. Round Duct Results, IL_{max} by round duct diameter

Low Frequency: For the low frequency data, the best fit line does not describe the data well, with an associated R^2 of only 0.0825. However, it appears that the dependent variable does not have a very large effect on the low frequency IL_{max} result. Again, the low frequency IL_{max} appears constant with regards to diameter.

The treatment levels appeared to have unequal variances, so the O'Brien's test was conducted on the group variances to determine whether they were sufficiently equal (homoscedasticity). The O'Brien test returned a p-value of 0.3893 for the low frequency data set, indicating homoscedasticity, but observation of the data in Figure 13 still indicated possible heteroscedasticity. Also, the Levene's test returned p=0.0257, indicating heteroscedasticity. Thus, although the data appear to have unequal variances, they may be considered adequately homoscedastic. ANOVA tests on the data returned p=0.7751 (no significant effect from diameter). In case of actual heteroscedasticity, Welch's ANOVA was also run on the data to confirm the normal ANOVA results. Welch's ANOVA returned p=0.4328 for the low frequency data with the same conclusions.

High Frequency: The high frequency data were certainly affected by the diameter. As the diameter increases, the cut-on frequency of the first higher order mode decreases, so that the overall IL_{max} at the higher frequencies should decrease as well. That is reflected in the data in Figure 11. A linear regression line fit to the data has an R^2 of 0.6872 and does describe the data relationship somewhat.

The high frequency data appeared to have unequal variances, since the data at 4 and 8 inches are much more tightly grouped than other treatments. Both the O'Brien and Levene tests on the high frequency data returned acceptable p-values of 0.5052 and 0.1150, respectively. The ANOVA test gave $p < 0.0001$ for the high frequency data. Further, the model for the high frequency data accounted for 90% of the variability in the data according to the ANOVA. In case of actual heteroscedasticity, Welch's ANOVA returned $p=0.0024$ for the high frequency data, which seemed to agree with the normal ANOVA results.

The null hypothesis that there was no effect from the cross-sectional width on the low frequency data could not be rejected at $\alpha=0.05$, while it was rejected for the high frequency data. It is important to recall that the low and high frequency groups are adjacent.

Round Duct Discussion

The estimated cut-on frequencies for the first higher order modes of the five duct sizes calculated from Equation 1 are presented in Table 4, along with the estimated dominant frequencies below which plane waves will dominate the noise signal, calculated from Equation 2. These frequencies were expected to be the upper boundary for the effective range for the simple ANC system (see Figure 6). As duct diameter increases, the upper boundary of plane wave domination decreases and IL_{max} at higher frequencies should correspondingly decrease.

Table 4. Estimated first higher order mode cut-on frequencies and maximum plane wave dominant frequencies by duct diameter

Diameter (inches)	Cut-on Frequency (Hz)	Maximum Plane Wave Dominant Frequency (Hz)
4	1695	1987
6	1130	1325
8	847	993
12	565	662
18	377	442

The results of the round duct study support the logic based on the higher order mode cut-on frequency Equation 1. For low frequencies below the first higher order mode cut-on frequency of the largest diameter (377 Hz), there was no effect of changing the diameter. In effect, all diameter systems operated on plane waves for the low frequency IL_{max} data. For high frequencies, there was a significant effect of changing the diameter, so that IL_{max} decreased with increasing diameter.

Round Duct Conclusion

For the high frequency IL_{max} data, there was a 1.4 to 11.1 dB average increase in insertion loss for successively smaller diameters (12, 8, 6, and 4 inches) compared to the 18 inch diameter duct. There is a significant effect of diameter on high frequency IL_{max} for round ducts that follows the logic of the underlying equation.

Overall Discussion

The p-values and summary results from all of the studies are displayed in Table 5.

Table 5. p-values and summary results from all studies to describe independent variables

				p-values and IL_{max} difference	
Study	**Variable**	**H_0**	**Test**	**Low Frequency**	**High Frequency**
Ia	Rectangular dimension	No effect of horizontal dimension	1-way ANOVA	p=0.3348 (no effect)	p=0.0220 (16.2-14 dB)
Ib	Round diameter	No effect of diameter	1-way ANOVA	p=0.4328 (no effect)	p=0.0024 (9.5 dB)

Reducing the cross-sectional dimensions of both rectangular and round ducts increases the frequency range of effective ANC IL. The overall ANC IL levels were higher for round ducts (18 to 29 dB for high frequency) than for the rectangular ducts (13 to 29 dB for high frequency). However, the main reason for the lower ANC IL values in the rectangular ducts could very probably be that one dimension remained fixed at 61 cm. That would mean that the first higher order mode would cut-on for all the variable width sizes at the beginning of the high frequency range.

Overall Conclusion

The application of this research to industrial noise control problems would be most useful for environmental noise from exhaust stack situations. However, decreasing duct size to improve insertion loss at higher frequencies may bring up other problems. One concern would be the increase in pressure requirements to the exhaust fan. For instance, if an 18 inch diameter duct were to be replaced with smaller ducts of equal total cross-sectional area (see Figure 3), the number of ducts would increase quickly. Table 6 lists the duct diameters used in the round duct study to increase high frequency ANC insertion loss, with the number of ducts of that size needed to equal the same cross-sectional area of a single 18 inch diameter duct. The increase in fan pressure from twenty 4 inch diameter ducts would be large (about $(D_1/D_2)^{1.22} = (18''/4'')^{1.22} = 6.26''$ w.g. for each duct) (Guffey and Hickey, 1983). This is not to mention the increase in cost simply to purchase and install 20 ducts. Further, the ANC hardware costs would become considerable to have a separate control channel for each duct.

Table 6. Number of ducts needed to equal the cross-sectional area of an 18 inch diameter duct

Duct diameter Studied (in)	# ducts needed for 18" area
4"	20
6"	9
8"	5
12"	2

Active noise control has long been an area of interest for acousticians and noise control engineers. The hardware limitations and need for expertise in implementation have limited the industrial applications of ANC technology. One application to which ANC is particularly well-suited is noise control on exhaust stacks. ANC works well to reduce the low frequency "rumble" that can travel great distances and annoy neighboring communities. However, for broadband noise sources, even a combination of active and passive controls may fall short of complete broadband noise control. This research indicated that the use of smaller ducts can extend the frequency range of control of ANC methods to higher frequencies. By providing up to 26 to 28 dB of insertion loss at 500 Hz, ANC may become more viable as an option for industrial noise control issues.

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